DOME PREDICTIONS FOR AN EQUATORIAL TELESCOPE

Patrick Wallace

Tpoint Consulting UK

ptw@tpointsw.uk

2017 June 10

1 Introduction

As an equatorial telescope points and tracks, the dome in which it is housed needs to rotate, and any windscreen arrangements need to be raised and lowered. If the telescope and the mount axes are all coincident, and located at the center of the dome, the calculation is trivial, simply the standard transformation from (h, δ) into azimuth and elevation (A, E). However, if misalignments and offsets are present, some additional coordinate geometry is required. The present document outlines an algorithm for dealing with this more general case.

2 Prerequisites

The given quantities are as follows:

ϕ	elevation of north end of polar axis
r_D	radius of dome
(x_m, y_m, z_m)	offset of mount in dome
p	separation of polar and declination axes
q	distance along declination axis
r	separation between telescope and declination axis
(h, δ)	telescope mechanical hour angle and declination

3 Context

1. The object of the exercise is to predict the point on the dome, azimuth and elevation (A, E), through which the telescope's optical axis passes. This depends on where the telescope is pointed, where within the dome the mount is located, the mount geometry, and where on the mount the telescope is fixed.

2. For the (h, δ) values, relatively low precision will usually suffice. For example the pointing model (corrections for the effects of minor flexures and misalignments *etc.*) can safely be ignored. However, it may be worth correcting for zero-point errors.

3. The attitude of the polar axis is given by ϕ, which is the elevation above the horizon of the north end of the axis, *i.e.* the latitude. *n.b.* For the southern hemisphere the value is negative, because the northern end of the polar axis is below the horizon.

4. The dome is presumed to be hemispherical (or some other portion of a sphere). For the radius of the sphere r_D, any desired units can be used as long as all the other "length" arguments are in the same units. In other words, the units of $_D$, (xm, ym, zm), p, q, and r must all be the same.

5. The vector (x_m, y_m, z_m) specifies the offset of the mount from the center of the dome. The "mount" in this context is that point along the polar axis (and hence fixed in space) that lies nearest to the declination axis. The (x, y, z) coordinate system is oriented (east,north,up).

6. The quantities p, q and r form a sequence of offsets linking the polar axis, the declination axis and the telescope's optical axis:

   - p is the separation between the polar and declination axes at their closest approach. For most GEMs (German equatorials) the two axes intersect and hence p is zero. Occasionally (some horseshoe mounts for example) the two axes do not intersect and p is non-zero.
   - q is the distance along the declination axis to where the telescope assembly turns, starting from the point on the declination axis closest to the polar axis. For fork mounts *etc.*, q is zero, but for GEMs q is a substantial distance.
   - r is the separation between the declination and optical axes. It is usually zero. Very occasionally, for example where a second telescope is mounted on the "side" of the main telescope, the declination axis and optical axis do not intersect and so r is non-zero.

7. The sign conventions for p, q and r are such that with the telescope pointing at (h, δ) coordinates $(0, 0)$, p is positive towards $(12^{\rm h}, 0)$, q is positive towards the east, and r is positive towards the north celestial pole (*n.b.* even in the southern hemisphere).

8. The offsets p and q are at right angles, and so are the offsets q and r, but the angle between the offsets p and r varies with declination.

9. The dome (A, E) that the algorithm delivers follows the normal convention. Azimuth A increases clockwise from zero in the north, through 90° ($\pi/2$ radians) in the east.

10. The telescope (h, δ) must be *mechanical* rather than *celestial*, so that above/below pole and east/west of the pier cases are distinguished. When the mount is in the "below the pole" configuration, the declination value δ will lie outside the range $\pm\pi/2$. In the GEM case this will happen half the time, because of the pier reversal necessary as targets cross the meridian.

4 Algorithm

First we calculate the vector mount to optical center, in the east-north-up coordinate system:

$$
\begin{aligned}
y &= p + r\sin\delta, && (1)\\
x_{mo} &= q\cos h + y\sin h, && (2)\\
y_{mo} &= -q\sin h + y\cos h, && (3)\\
z_{mo} &= r\cos\delta. && (4)
\end{aligned}
$$

Next the vector dome to optical center, again in the east-north-up frame:

$$
\begin{aligned}
x_{do} &= x_m + x_{mo}, && (5)\\
y_{do} &= y_m + y_{mo}\sin\phi + z_{mo}\cos\phi, && (6)\\
z_{do} &= z_m - y_{mo}\cos\phi + z_{mo}\sin\phi. && (7)
\end{aligned}
$$

The telescope (A, E) unit vector, same frame:

$$
\begin{aligned}
x &= -\sin h\cos\delta, && (8)\\
y &= -\cos h\cos\delta, && (9)\\
z &= \sin\delta, && (10)\\
x_s &= x, && (11)\\
y_s &= y\sin\phi + z\cos\phi, && (12)\\
z_s &= -y\cos\phi + z\sin\phi. && (13)
\end{aligned}
$$

We now solve for the distance from the optical center to the dome aperture:

$$
\begin{aligned}
s_{dt} &= x_s x_{do} + y_s y_{do} + z_s z_{do}, && (14)\\
t_m^2 &= x_{do}^2 + y_{do}^2 + z_{do}^2, && (15)\\
w &= s_{dt}^2 - t_m^2 + r_D^2, && (16)\\
f &= -s_{dt} + \sqrt{w}. && (17)
\end{aligned}
$$

If the value of w calculated in (16) is negative, there is no solution.

Almost there! The vector from the dome center to the dome aperture is simply...

$$
\begin{aligned}
x &= x_{do} + f x_s, && (18)\\
y &= y_{do} + f y_s, && (19)\\
z &= z_{do} + f z_s. && (20)
\end{aligned}
$$

...and all that remains is to convert to spherical coordinates:

$$
\begin{aligned}
A &= \arctan x/y, && (21)\\
E &= \arctan z/\sqrt{(x^2 + y^2)}. && (22)
\end{aligned}
$$

Here *arctan* is assumed to return an all-quadrants result following the usual `atan2` convention. The special case of $x = y = 0$ needs to be trapped and $A = 0$ (say) returned. It is probably best to return A in the range $0 - 2\pi$.

5 Example

For this example...

- The mount is a GEM and is at latitude $+36.18°$.
- The dome is 3.8 meters in diameter.
- The optical axis and the declination axis intersect.
- The declination axis and the polar axis intersect.
- When the mount is set to $(0, 0)$, the telescope is east of the pier and the counterweight is west of the pier.
- The distance from the polar axis along the declination axis to the optical axis is 505 mm.
- The point at which the polar axis and declination axis intersect is 35 mm west of, 370 mm north of, and 1250 mm above the center of the dome.
- The telescope is pointing at a star 10 minutes west of the meridian, at declination $+37.9°$, and is east of the pier.

...the inputs are:

- $\phi = +0.6315$ (elevation of the north end of polar axis in radians)
- $r_D = 1900.0$ (radius of the dome in mm)
- $x_m, y_m, z_m = -35.0, +370.0, +1250.0$ (offset of mount from dome)
- $p = 0.0$ (separation of mount axes)
- $q = 505.0$ (distance of telescope along declination axis)
- $r = 0.0$ (separation of declination and optical axes)
- $h = +0.0436$ (telescope mechanical hour angle in radians)
- $\delta = +0.6615$ (telescope mechanical declination in radians)

...and the predicted (A, E) is:

$$(50.369411°, 72.051742°).$$

To acquire the same star but with the telescope west of the pier would require $h = -3.098$ radians and $\delta = 2.480$ radians, and the prediction becomes:

$$(305.595067°, 68.824495°)$$

(To assist in testing, the predictions have been quoted to more decimal places than is justified by the inputs.)